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The Cavendish Experiment as a Tool for Historical Understanding of Science

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Abstract Following an ever growing literature which takes serious the relevance of case-studies in the history of science for science education and understanding of science, I provide a detailed historical reconstruction of the Cavendish Experiment, which remains as close as possible to the original. In this paper, I call attention to three educational benefits of familiarizing students with the Cavendish Experiment and its aftermath.

1 The Cavendish Experiment in an Educational Context

In this paper, which is primarily intended for teachers and lecturers in Physics and History of Science, it will be shown that much can be gained from a historically sensitive reconstruction of the Cavendish Experiment in an educational context.[1] When Henry Cavendish (1731–1810), who according to Russell McCormmach was "the first after Newton to possess mathematical and experimental talents at all comparable to Newton's" (*DSB*, III, p. 195), was nearly 67, he published what would become his last substantial scientific paper: 'Experiments to Determine the Density of the Earth,' which appeared in 1798. For reasons that will be explained in what follows, Cavendish never conceived of his experiment as an attempt to measure the gravitational constant (Moreno Gonzáles 2001; Clotfelter 1987; Lally 1999; Jungnickel and McCormmach 2001, p. 444, footnote 87), despite numerous accounts in Physics textbooks to the contrary.[2] Since no draft material

[1] Useful discussion of the Cavendish experiment is to be found in Titchmarsh (1966), McCormmach (1995, 1998), Falconer (1999), Jungnickel and McCormmach (2001, pp. 440–450), and Lauginie (2007). However, it must be noted that most of the above accounts are not very detailed when it comes to the specifics of Cavendish' calculations. In discussing Cavendish' results, I shall preserve his original mathematical reasoning, which appears somewhat archaic in comparison to our contemporary tools of mathematical computation.

[2] A characteristic example states: "The universal gravitational constant *G* was first measured in an important experiment by Sir Henry Cavendish in 1798." (Serway and Jewett 2006, vol. I, p. 338).

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connected to Cavendish' famous experiment has surfaced so far and it is unlikely that it ever will (McCormmach 1995, p. 22), the only source at our disposal is Cavendish' published account of his experiments with the torsion rod (Cavendish 1798). However, Cavendish' technical paper, in which he ingeniously determined the mean density of the earth, requires hard work for lecturers and their students. In order to facilitate teaching of the Cavendish Experiment, it is my aim in this paper to unearth both the general features of Cavendish' tackle on the matter as well as the technicalities involved. By providing a careful reconstruction of the Cavendish Experiment, I seek to align myself to the growing literature which takes historical case-studies serious as adequate tools for science education and understanding of science.[3]

What are the educational benefits of teaching a detailed reconstruction of the Cavendish Experiment that will be focused on in this paper?[4] Here I shall focus on three important benefits, which will be explained in more detail below:

(a) The Cavendish Experiment is a vivid illustration of how *a previously disconnected or "imponderable"*[5] *physical quantity*, i.e. the mean density of the earth, *was inferred from the gravitational interaction between laboratory-sized bodies*.
(b) The Cavendish Experiment is a paradigmatic case of *how conclusions about the empirical world were obtained in the absence of equations*.
(c) The Cavendish Experiment and its aftermath provide an ideal occasion to *highlight the evidential significance of so-called 'normal science'*.

I shall begin by discussing (a) and (b) jointly and by making some suggestions as to how students, who have already been familiarized with the theory of universal gravitation, might be introduced to the Cavendish Experiment. For such students, calculating the density of the earth might not appear to be much of a challenge. In fact, given the theory of universal gravitation, it is pretty straightforward. As a stage-setting for a historical discussion of the Cavendish Experiment, it could be very useful to invite the students to calculate the mean density of the earth for themselves. After some manipulation of equations, they will very likely come up with the following train of reasoning. From the definition of density, we know that the density of the earth, $\rho(e)$, equals $\frac{m(e)}{V(e)}$, where $m(e)$ is the mass of the earth and $V(e)$ its volume. Furthermore, from the definition of density and from $F = \frac{G \cdot m(e) \cdot m'}{R(e)^2} = g \cdot m'$, where $R(e)$ is the radius of the earth and m' is the mass of a body near the surface of the earth, it follows that $\rho(e) = \frac{g \cdot R(e)^2}{G \cdot V(e)}$ (*). If we, furthermore, approximate the figure of the earth as a sphere, it follows that the volume of the earth, $V(e)$, equals $\frac{4}{3} \cdot \pi \cdot R(e)^3$ (**). When we fill in this determination of the earth's volume in (*), we obtain: $\rho(e) = \frac{g \cdot R(e)^2}{G \cdot \frac{4}{3} \pi \cdot R(e)^3} = \frac{g}{G \cdot \frac{4}{3} \pi \cdot R(e)}$. Since we have reduced all involved quantities to known

Footnote 2 continued
Unfortunately, the myth of Cavendish and big *G* has also persisted in the scholarly literature on the matter (Kuhn 1996, pp. 27–28; Baigre 1995, pp. 113–116).

3 For example, Chang (2011), Chavicchi (2008), Conant (1957), Heering (2000), Höttecke (2000), Martinez (2006), Metz and Stinner (2006), and Palmieri (2008).

4 Obviously the features that will be explored here are not meant as exhaustive. In Lauginie (2007), the educational significance of the Cavendish experiment has been highlighted as well. However, the features of the Cavendish experiments I want to call attention to here are rather different than those focused on in Lauginie (2007, pp. 130–142). I therefore consider the paper at hand and Lauginie (2007) as complementing each other.

5 Hacking's terminology (Hacking 1983, p. 236).

ones, our problem is solved. How could it be, the students might wonder, that it took scientists more than a century after the publication of the theory of universal gravitation (1687) to arrive at this straightforward calculation? Additionally, they might even question whether the Cavendish Experiment was really all that ground-breaking.

At this point, the students should be introduced to some generalities surrounding the complex historical context of the Cavendish Experiment. To begin with, it should be brought to their attention that, since the first measurements of the gravitational constant, G, only appeared at the end of the nineteenth century (Poynting 1892; Boys 1895; Braun 1896; Richarz and Krigar-Menzel 1898; see Sect. 3.2), it was impossible for Cavendish to rely on G in calculating the mean density of the earth from the theory of universal gravitation. What is really crucial is that they are shown, moreover, why Cavendish could not conceive of the notion of a gravitational constant within his physics. For this purpose, it should be brought to their attention that, just as Newton, Cavendish worked entirely within a mathematical framework based on *proportions*, whereas the constant of universal gravitation can only be conceived within a mathematical framework of *equations and absolute measurements*, as I shall explain in what follows. Propositions 7 and 8 of Book III of the *Principia* jointly establish the law of universal gravitation, but, just as all propositions in the *Principia a calce ad capitum*,[6] this law was expressed verbally in terms of proportions:

> **Proposition 7** *Gravity exists in all bodies universally and is proportional to the quantity of matter in each*. (Newton 1999, p. 810)
>
> **Proposition 8** *If two globes gravitate toward each other, and their matter is homogeneous on all sides in regions that are equally distant from their centers, then the weight of either globe toward the other will be inversely as the square of the distance between the centers*. (ibid., p. 811)[7]

In other words, in Newton's *Principia* no trace can be found of the *equation* $F = \frac{G \cdot m \cdot m'}{r^2}$. In this context, the role of G, i.e. $6.674 \times 10^{-11}\ N\left(\frac{\mathrm{m}^2}{\mathrm{kg}^2}\right)$, in the formula of universal gravitation should be explained to the students. The second part of the units of G, $\frac{\mathrm{m}^2}{\mathrm{kg}^2}$, cancels out the units of the other physical quantities in the right side of formula of universal gravitation, $\frac{\mathrm{kg}^2}{\mathrm{m}^2}$, so that only N remains. This is exactly what G does in the equation of universal gravitation: *it provides a unit of measure of force*, N, which is crucial if we want absolute measurements of gravitation. Before the end of the nineteenth century, gravitational interaction was not measured in standard units. Correspondingly, determinations of the mean density of the earth based on the theory of universal gravity were not given in absolute terms, rather the density of the earth was measured proportionally, i.e. relative to the density of water. Historically, it was thus not the case that Cavendish manipulated some *equations* in order to calculate the mean density of the earth, rather he sought to establish a *proportion* by which the density of the earth could be determined relative to the density of water.

By contrasting absolute measurements with measurements in terms of proportions, the students will gain more insight into the historicity, specificity and advantages of an equation-based physics. Emphasizing the differences between how physics is done now and how it was done in the past, will help to stir their interest in Cavendish' accomplishment. They will begin to wonder how is was that Cavendish succeeded in finding a proportion between the density of the earth and the density of water on the basis of the

[6] For similar observations on Galileo's mathematics, see Palmieri (2003, p. 230ff).

[7] For the details of how Newton arrived at these proportions, I refer the interested reader to Ducheyne (in press), Chapter 3, 3.4.5.

gravitational interaction of laboratory-sized bodies. It is at this stage that they will be ready for a reconstruction of the actual Cavendish Experiment. In Sect. 2, I shall provide a detailed historical reconstruction of the Cavendish Experiment by clarifying *what Cavendish measured*, *how he measured it*, and *how he determined the density of the earth relative to the density of water*, while staying as close as possible to Cavendish's original calculation, as Höttecke (2000) advises. My reconstruction of the Cavendish experiment is written for the purpose of highlighting educational benefits (a) and (b) as good as possible.

Let us, finally, turn to (c). Newton's theory of universal gravitation was well-confirmed at planetary distances and, to a significant extent, it was founded on astronomical observations (Kuhn 1996, p. 31). By means of his experiment, Cavendish had not merely determined the mean density of the earth, at the same time, he had tested whether the law of universal gravitation breaks down at terrestrial distances and, in doing so, he had shown that *robust*[8] gravitational interactions occur between laboratory-sized bodies.[9] In Sect. 3, which is written for the purpose of highlighting educational benefit (c) as good as possible, I shall focus on the aftermath of the Cavendish Experiment. I shall highlight that generations of physicists after Cavendish succeeded in establishing *increasingly stronger* evidence for the robustness of the gravitational interaction between laboratory-sized bodies, and, hence, for the *universality* of Newton's theory of gravitation. As I shall explain there, Thomas S. Kuhn denied the evidential significance of so-called 'normal science'. In close opposition to Kuhn, I shall call attention to the evidential significance of 'normal science'. At the same time, I shall emphasize the educational benefits of doing so.

2 The Cavendish Experiment as a Case-Study

This section is divided into four subsections: in Sect. 2.1, I provide a description of Cavendish' apparatus, while at the same time highlighting some of its salient characteristics; in Sect. 2.2, I discuss what and how Cavendish measured with his apparatus; in Sect. 2.3, I analyze Cavendish' mathematical train of reasoning, which enabled him to establish a proportion for the mean density of the earth relative to the density of water; and, in Sect. 2.4, I discuss Cavendish' computation of the mean density of the earth.

2.1 Cavendish' Apparatus and its Characteristics

A thorough presentation of Cavendish' apparatus is a natural starting point for any reconstruction of the Cavendish Experiment. I shall provide a description of the apparatus based on Cavendish (1798, pp. 469–473), and, afterwards, I shall emphasize some of its salient features.

Cavendish' experimental device was a refinement of an apparatus originally contrived by John Michell (1724–1793) (McCormmach 1968; McCormmach in press), who "did not complete the apparatus till a short time before his death, and did not live to make any experiment with it" (Cavendish 1798, p. 469). After Michell's death it came into the hands

[8] In the sense that they were shown to be *independent* from the surrounding variations in temperature or air currents (Galison 1987, pp. 2–3).

[9] Obviously, Cavendish did not provide a test for the gravitational inverse-square law (Lauginie 2007, pp. 126–127).

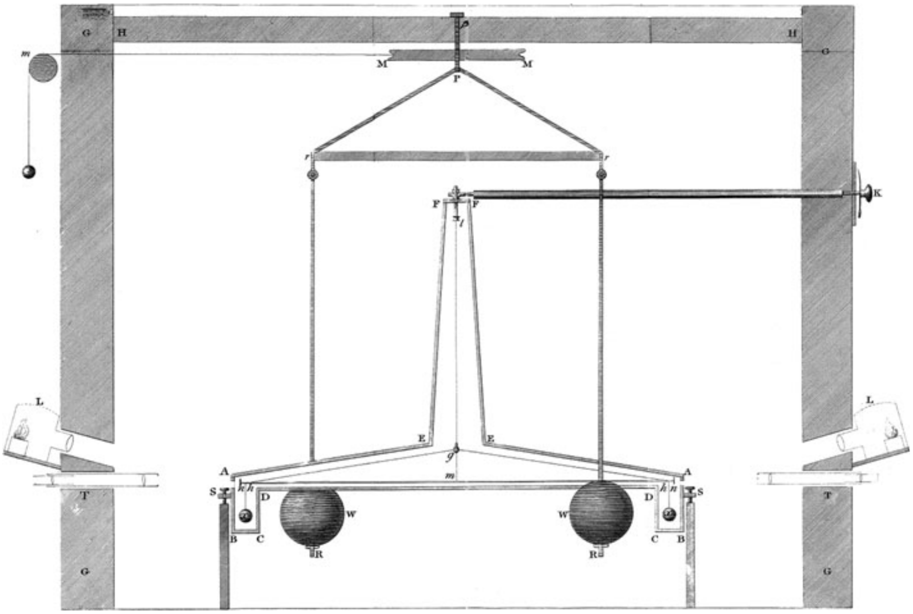

Fig. 1 Cavendish' experimental apparatus (longitudinal vertical section). Taken from Cavendish (1798, p. 526). Courtesy of The Royal Society

of Francis John Hyde Wollaston, the Jackonian Professor at Cambridge (*DSB*, XIV, pp. 484–486), who, Cavendish wrote, "not having conveniences for making experiments with it, in the manner he could wish, was so good as to give it to me" (Cavendish 1798, p. 469; see, furthermore, Jungnickel and McCormmach 2001, pp. 441–442). Figure 1 shows a longitudinal vertical section through the apparatus and the surrounding room.

In order to guard against sources of error, the room, measuring 10 feet in height and as many feet across, remained shut throughout the experiment and the effects were observed from outside of the room by means of telescopes (*T*) and lamps (*L*), which were installed at both sides of the room and which pointed to the verniers placed inside the case (Cavendish 1798, p. 471). In this way, the most significant source of error, namely variations of temperature, could be guarded against significantly, according to Cavendish (ibid., p. 471). Two leaden balls *x* and *x*, which have a diameter of about two inches (or about 5.08 cm), were suspended by the wires *hx* from the arm *ghmh*, which is itself suspended by the slender wire *gl* with a length of about 40 inches (or 1.016 m). Given the fact that the wire is sufficiently slender, "the most minute force, such as the attraction of a leaden weight a few inches in diameter, will be sufficient to draw the arm sensibly aside" (ibid., p. 469). To determine the force by which the balls and the arm are drawn against the restoring force of the twisted wire, the arm was placed in such a way so as to enable it to move freely as a 'horizontal pendulum,' which is, as we will see in Sect. 2.3, vital to Cavendish approach. Using a horizontal pendulum also simplifies matters experimentally: because the balls are set in a plane orthogonal to the direction of the earth's gravitational field, Cavendish succeeded in neutralizing gravitation's downward pull on the oscillating balls. The arm *ghmh*, measuring 6 feet (or roughly 1.83 m) consisted of a slender deal rod *hmh* strengthened by a silver wire *hgh*, which "is made strong enough to support the balls,

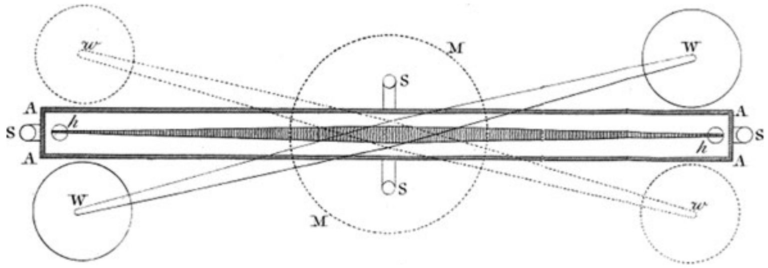

Fig. 2 Cavendish' experimental apparatus (view from above). Taken from: Cavendish (1798, p. 527). Courtesy of The Royal Society

though very light" (ibid., p. 472).[10] The two lead balls *x* and *x* are placed in the narrow wooden case *ABCDDCBAEFFE*, which is installed horizontally and which is supported by posts fixed firmly in the ground to which it is attached to by four screws (*S*).[11] The wooden case served to protect the arm from air currents. *FK* represents a wooden rod, which, by means of an endless screw, turns around the support and to which the slender wire *gl* is fastened. By means of *FK* Cavendish could manipulate the position of the arm *ghmh* from outside the room till the arm settles in the required position without any danger of touching either side of the case. The wire *gl* is fastened to its support at the top and to the center of the arm at the bottom by brass clips in which it is pinched by screws. From the copper rods *Pr* and *rR* and the wooden bar *rr*, which is placed above the rod, two lead weights *W* and *W* are suspended at the same height as the balls. This device was attached to the center pin *Pp* which was attached to the ceiling *HH* of the room and placed above the center of the apparatus. To *Pp* the pulley, *MM*, around which the cord *Mm* was attached so that one can alter the position of the weights *W* and *W* from outside the room.[12]

When the weights *W* and *W* were in the first position—indicated by full lines—they conspired in drawing the arm in the direction *hW*; when the weights are in the second position—indicated by dotted lines—they attracted the arm in the contrary direction *hw* (see Fig. 2, which depicts a view from above of the instrument). Because in the second position the arm was drawn aside in such a direction as to make the index point to a higher number on the ivory slips, Cavendish considered this as the "positive position of the weights." The weights *W* and *W* were furthermore prevented from striking the instrument by pieces of wood, fastened to the wall of the room, which stop the weights as soon as they come within one fifth of an inch (or 0.508 cm) of the case. Cavendish found that "the weights may strike against them with considerable force, without sensibly shaking the instrument" (ibid., p. 473). Moreover, "[i]n order to determine the situation of the arm" (ibid., p. 473), slips of ivory, which were divided to a twentieth of an inch (or 1.27 mm), were place within the

[10] In an accompanying footnote, Cavendish pointed out that this set-up is easier to construct, meets less air resistance and involves less complicated computations to ascertain how much the rod was attracted by the weights.

[11] In Fig. 1, the longitudinal vertical section of the apparatus, only two of the four screws are depicted. All four screws are depicted on Fig. 2. Cavendish noted that the box in which the balls are moved is pretty deep "which makes the effect of the current of air more sensible than it would otherwise be, and is a defect which I intend to rectify in some future experiments" (Cavendish 1798, p. 497).

[12] Jungnickel and McCormmach note that in these experiment Cavendish "brought the earth into his place of privacy, his home [in an outhouse of Clapham Common], where he experimented on it on his own" (Jungnickel and McCormmach 2001). On Cavendish' personality traits, see Jungnickel and McCormmach (2001, pp. 303–309); on Clapham Common, see (ibid., pp. 324–331).

case, as near to each end of the arm as could be possibly done without touching them. To the original slips on each side a nonius was added, which in its turn was divided into five parts so that the position of the arm could be measured to one 100th of an inch (i.e., to 0.254 mm). Once the arm is set to rest and its position was observed, Cavendish moved the weights *W* and *W* closer to the balls *x* and *x* so that "the arm will not only be drawn aside thereby, but it will be made to vibrate, and its vibrations will continue for a great while" (ibid., p. 474).

In an educational context, it is worthwhile to emphasize some salient features of Cavendish' apparatus. Since G equals $6.674 \times 10^{-11}\ N\left(\frac{m^2}{kg^2}\right)$, gravitational forces are very weak.[13] Therefore, gravitational experiments in the laboratory are *highly susceptible to extraneous disturbances* (Chen and Cook 1993, xii, p. 5, pp. 34–57). Measuring gravitation in the laboratory is therefore far from unproblematic and to the present day this difficulty persists (Gillies 1997).[14] In Cavendish' experimental set-up, extraneous disturbances were guarded against significantly: in order to guard against *temperature variations*, Cavendish installed the apparatus inside a small room, and, in order to guard against *air currents*, he placed the torsion rod in a wooden case. Given these screening-off procedures, Cavendish succeeded in arriving at a quite accurate value for the mean density of the earth, as will be shown in Sect. 2.4.

That the Cavendish Experiment involved known masses is, furthermore, highly significant—despite its obviousness in present-day physics curricula. Prior to the Cavendish Experiment with the torsion rod, different attempts to determine the density of the earth were undertaken. A well-tried method consisted in measuring the deflection of a plumb line in the vicinity of a large mountain.[15] This was the method which was used in Nevil Maskelyne's and Charles Hutton's famous experiment at Mount Schiehallion in Scotland (Maskelyne 1775; Hutton 1779).[16] In a short but acutely written paper, W. S. Jacob pointed out that "the Cavendish experiment is the one which may be relied on as giving a good approximation to the truth, within limits or error (when conducted with proper precaution)" (Jacob 1857, p. 295). In the Cavendish experiment "we are dealing with disturbing masses whose amount is exactly known," whereas in the method promoted by Hutton (and Maskelyne) "we may approximately measure the mass of the mountain *above* the surface, we do not know how much may be added or abstracted *below*; and we have no right to assume that the mountain is merely a detached mass resting upon the general surface; it will almost certainly have *roots* differing in density from the surrounding country" (ibid., pp. 297–298, cf. Pointing 1892, p. 621).[17] Because of the high uncertainty of their approximation of the mass of the mountain, the method promoted by Maskelyne and

[13] Cavendish computed that the force by which the balls are attracted in proportion to their weights is as 1 to 50,000,000 (Cavendish 1798, p. 470).

[14] Note that the precision with which the inverse-square law can be established is about one part in 10^4, whereas that of the inverse-square law in electrostatics is about one part in 10^{16}.

[15] This method seems to have been tried for the first time by Pierre Bouguer (1698–1758) (Bouguer 1749, pp. 372–373). See furthermore Howarth (2007, pp. 230–231). On Bouguer, see (*DSB*, II, pp. 343–344).

[16] Cavendish was involved in the mathematical parts of Maskelyne's and Hutton's experimental work on the matter (Jungnickel and McCormmach 2001, pp. 259–261). On the Mount Schiehallion expedition, see Danson (2005, pp. 106–154).

[17] Nineteenth-century experiments, which sought to determine the mean density of the earth on the basis of the observed difference between the rates of two invariable pendulums (one at the surface of the earth and the other at the bottom of a deep shaft), were faced with related problems (Airy 1856; Sterneck 1883; for discussion, see Ducheyne 2011, pp. 204–205). In calculating the density of the earth from such experiments, an estimate had to be introduced about the mean density of the outer spherical shell of the earth, which has a thickness *x* which is equal to the depth of the shaft.

Hutton was unsuccessful in controlling the physical quantities involved and for this reason it was ultimately abandoned.

2.2 What and how Cavendish Measured

Once the students have been familiarized with the *modus operandi* of Cavendish' apparatus, it is time to explain to them how it was used as a measuring device. It is relevant to note that Cavendish was a careful experimenter who was very knowledgeable of the calibration of scientific instruments—in fact, he was very active at times when the instruments of the Royal Society were being calibrated (Jungnickel and McCormmach 2001, p. 149, p. 174, pp. 220–224). Crucial to the Cavendish Experiment was the measurement of the *time of a vibration* and of the *motion of the arm.*

As has been highlighted in Sect. 2.1, Cavendish could measure the motion of the arm to 1/100 of an inch. His measurement of the time of a vibration, however, requires additional discussion and, more precisely, some background on two crucial concepts: the *point of rest (of a vibration)* and the *time of (a) vibration*. To establish the point of rest, it was necessary "to observe the extreme points of the vibrations, and from thence to determine the point which it would rest at if its motion was destroyed, or the point of rest, as I shall call it" (Cavendish 1798, p. 474). For this purpose, Cavendish observed the first three successive extreme points of vibration and took the mean between the first and third of these extremes as the extreme point of vibration (in one direction) and he took the mean of the extreme point of a vibration and the second extreme as the *point of rest*, "for *as the vibrations are continually diminishing*," he observed, "*it is evident, that the mean between two extreme points will not give the true point of rest*" (Cavendish 1798, p. 474 [italics added]). Consequently, if x_1, x_2 and x_3 are the first three successive extreme points of a vibration, the point of rest is given by: $\frac{1}{2}\left(\frac{x_1+x_3}{2}+x_2\right)$. It is important to explain to the students why Cavendish determined the point of rest from the first three extremes: because of the very long period of oscillation of the rod and very small damping, Cavendish could not wait hours for the rod to reach its equilibrium, which would be increasingly subject to external disturbances (Lauginie 2007, p. 124). As Cavendish pointed out himself:

> It may be thought more exact, to observe many extreme points of vibration, so as to find the point of rest by different sets of three extremes, and to take the mean result; but it must be observed, that notwithstanding the pains taken to prevent any disturbing force, *the arm will seldom remain perfectly at rest for an hour together; for which reason, it is best to determine the point of rest, from observations made as soon after the motion of the weights as possible*. (Cavendish 1798, p. 474 [italics added]).

He then determined the time of vibration by observing the two extreme points of a vibration and the times at which the arm arrived at two given divisions between the extremes, which were on different sides of the middle point and not very far from it. From the above, he computed the middle point of the vibration and, by proportion, the time at which the arm comes to this middle point. After a number of vibrations he repeated this procedure and divided the interval of time, between the arrival of the arm to the two middle points, by the number of vibrations, which gives *the time of one vibration.*[18] "To judge the property of this method," on must consider "in what manner the vibration is affected by the resistance of the air, and by the motion of the point of rest" (ibid., p. 476). Cavendish, however, argued that in both cases the effect will be inconsiderable. First, "as the time of

[18] Cavendish notes that the error in the result is much less, when the forces required to draw the arm aside was deduced from experiments made at each experiment, than when it is taken from previous experiments (Cavendish 1798, p. 478).

coming to the middle point is before the middle of the vibration, both in the first and last vibration, and in general is nearly so, the error produced from this cause must be inconsiderable." Secondly, insofar as the point of rest can be considered as moving uniformly, the time of two successive vibrations "will be very little altered; and, therefore the time of moving from the middle point of one vibration to the middle point of the next, will also be very little altered" (ibid., pp. 476–477).[19]

The Cavendish Experiment involved a total of 17 related experiments, which taken together make up for 29 separate observations (see Fig. 3).[20] In the first three experiments, Cavendish used a copper silvered wire, which, as he soon found out, was not stiff enough so that "the attraction of the weights drew the balls so much aside, as to make them touch the sides of the case" (ibid., p. 478). However, he decided to make some experiments with it. In order to make sure that the vibrations were not produced by magnetism, he changed the iron rods, by which the leaden weights were suspended, for copper ones, and a result of this it turned out that "there still seemed to be some effect of the same kind, but more irregular, so that I attributed it to some accidental cause, and therefore hung on the leaden weights, and proceeded with the experiments" (ibid., p. 479). With respect to the first three experiments, Cavendish remarked that "the effect of the attraction seems to increase, for half an hour, or an hour, after the motion of the weights; as it may be observed, that in all three experiments, the mean position kept increasing for that time, after moving the weights to the positive position; and kept decreasing, after moving them from the positive to the midway position." A hypothesis which could account for this is "that there might be a want of elasticity, either in the suspending wire, or something it was fastened to, which might make it yield more to a given pressure, after a long continuance of that pressure, than it did at first" (Cavendish 1798, p. 485). However, Cavendish observed that:

> if a wire is twisted only a little more than its elasticity admits of, then, instead of setting, as it is called, or acquiring a permanent twist all at once, it sets gradually, and, when it is left at liberty, it gradually loses part of that set which it acquired; so that if, in this experiment, the wire, by having been kept twisted for 2 or 3 h, had gradually yielded to this pressure, or had begun to set, it would gradually restore itself, when left at liberty, and the point of rest would gradually move backwards; but, though the experiment was repeated twice, I could not perceive any such effect. (ibid., p. 485)

In the experiments made thereafter, he replaced the original wire by a stiffer one. In the fourth experiment, Cavendish observed that, the effect of the weights seemed to increase on standing, in all three motions of the weights, similarly to what was observed with the former

[19] Baily's 1843 procedure for determining the times of vibrations was different from the one put to use by Cavendish: whereas Cavendish was happy to determine the time of a vibration for a whole series of changes in the positions of the masses for a single experiment—thereby assuming that the times of vibration are constant, Baily determined the time of vibration for every change of position of the masses (Baily 1843, p. 50, pp. 51–56). He used a similar procedure for determining the resting points (Baily 1843, p. 52). As Baily himself observed: "Cavendish always took the *second mean* of the extreme points as the true position of the resting point: and always compared his *last* true resting point in one experiment, with the *first* true resting point of the next succeeding experiment, for the purpose of determining the deviation […]. […] For Cavendish always continued the motion of the torsion rod for an indefinite period after the determination of the resting point for the deviation, and deduced the mean time of vibration from observations made at the beginning and end of that period: not perhaps bearing in mind that, during that period, the time of vibration might be (as, indeed, it often is) subject to change. Whereas, on the contrary, I have always considered the true time to be that which occurs during the motion of the very vibrations that are employed for determining the resting points; having had frequent experience of sudden changes in the time of vibration, without any apparent cause: which changes, though perhaps not always very great, might sometimes sensibly affect the results, if not carefully attended to" (Baily 1843, pp. 55–56).

[20] These were performed in 1797 on 6, 7, 12, and 20 August, 6, 18, and 23 September and in 1798 on 29 April, 5, 6, 9, 25–28, and 30 May. The paper was read on 21 June 1798.

Fig. 3 Summary of Cavendish' measurements. Taken from: Cavendish (1798, p. 520). Courtesy of The Royal Society

Exper.	Mot. weight	Mot. arm	Do. corr.	Time vib.	Do. corr.	Density.
1	m. to +	14,32	13,42	′ ″	-	5,5
	+ to m.	14,1	13,17	14,55	-	5,61
2	m. to +	15,87	14,69	-	-	4,88
	+ to m.	15,45	14,14	14,42	-	5,07
3	+ to m.	15,22	13,56	14,39	-	5,26
	m. to +	14,5	13,28	14,54	-	5,55
4	m. to +	3,1	2,95		6,54	5,36
	+ to −	6,18	-	7,1	-	5,29
	− to +	5,92	-	7,3	-	5,58
5	+ to −	5,9	-	7,5	-	5,65
	− to +	5,98	-	7,5	-	5,57
6	m. to −	3,03	2,9	-	-	5,53
	− to +	5,9	5,71		-	5,62
7	m. to −	3,15	3,03	7,4 by mean.	6,57	5,29
	− to +	6,1	5,9			5,44
8	m. to −	3,13	3,00		-	5,34
	− to +	5,72	5,54	-	-	5,79
9	+ to −	6,32	-	6,58	-	5,1
10	+ to −	6,15	-	6,59	-	5,27
11	+ to −	6,07	-	7,1	-	5,39
12	− to +	6,09	-	7,3	-	5,42
13	− to +	6,12	-	7,6	-	5,47
	+ to −	5,97	-	7,7	-	5,63
14	− to +	6,27	-	7,6	-	5,34
	+ to −	6,13	-	7,6	-	5,46
15	− to +	6,34	-	7,7	-	5,3
16	− to +	6,1	-	7,16	-	5,75
17	− to +	5,78	-	7,2	-	5,68
	+ to −	5,64	-	7,3	-	5,85

wire (ibid., p. 489). In the fifth experiment, the case was different: on moving the weights from positive to negative, the effect of increased on standing, yet, on moving them from negative to positive, it diminished. Next, he determined whether the balls or weights could have acquired polarity from the earth's magnetic field or whether magnets placed in the vicinity of the case could alter the observed effects (ibid., pp. 490–491). Upon closer scrutiny, this putative cause indicated no significant difference, according to Cavendish. He found, however, that differences in temperature did make a difference (sixth to eighth experiment) (ibid., pp. 496–497). Cavendish also pointed out that "the box in which the balls play is pretty deep, and the balls hang near the bottom of it, which makes the effect of the current of air more sensible than it would otherwise be, and this is a defect which I intend to rectify in some future experiments" (ibid., p. 497). Next, he compared the results when starting the experiment with the index placed very closely to the case without touching it (ninth to eleventh experiment), with the index in its usual position (twelfth to fourteenth experiment), and with the index placed very closely to the case without touching, but now in the opposite direction (fifteenth experiment). Two additional experiments concluded the observations.

In the two preceding paragraphs we have seen that Cavendish was not only concerned with making high-quality measurements, but also with investigating whether other causes, i.e. causes not due to gravity, were influencing the motion of the balls. We have also seen that Cavendish was clearly aware of the limitations of his apparatus, for he pointed out that the deepness of the box in which the balls are placed "makes the effect of the current of air more sensible than it would otherwise be." Both aspects underscore Cavendish' experimental skill. Based on his determinations of the motion of the arm (henceforth: *B*; see third and fourth column of Fig. 3) and the time of its vibrations (henceforth: *N*; see fifth and sixth column of Fig. 3), Cavendish would infer the mean density of the earth. In the following subsection, we will see how he came up with an ingenious way to determine the density of the earth relative to the density of water in terms of the observed values for the motions of the arm and the time of vibration.

2.3 Deriving the Mean Density of the Earth by Proportional Reasoning

As I have pointed out in the introduction, Cavendish' derivation of the mean density of the earth is a paradigmatic example of how a previously "imponderable" physical quantity was inferred on the basis of mathematical proportions. In this context, Russell McCormmach has pointed out: "Using modern terminology and notation this derivation can be done with few lines of equations, but they would not correspond to Cavendish's reasoning" (McCormmach 1995, pp. 15–16). Therefore, in order to fully understand the significance of the Cavendish Experiment, it is required that we study Cavendish' original train of reasoning, which was based on proportions, as close as possible. If, on the contrary, we approach the Cavendish Experiment with our contemporary physical concepts and mathematical techniques its significance will elude us. At this point, the students will have understood what Cavendish needed to provide: a mathematical relation in which the means density of the earth is given in terms of the motion of the arm and the time of vibration. There are three relevant steps in Cavendish' argument.

> *Step 1*: Solving the proportion of the force which must be applied to each ball to draw the arm aside by one scale division to the force of gravity on each ball in terms of the period of vibration of the arm.

First of all, Cavendish determined the force required to draw the arm aside, which is determined by the time of a vibration. He treated the motion of the arm as *a horizontal pendulum* which he compared to the motion of a regular (vertical) pendulum. Given the theoretical similarity between them, Cavendish was able to transfer certain proportions, which hold for a vertical pendulum, to the horizontal pendulum at hand. Because the distance between the centers of the two balls, *x* and *W*, is 73.3 inches, the distance of each from the center of motion is 36.65 inches. Moreover, the length of a pendulum vibrating seconds "in this region" is 39.14 inches. Therefore,

> if the stiffness of the wire by which the arm is suspended in such, that the force which must be applied to each ball, in order to draw the arm aside by the angle *A*, is to the weight of that ball as the arch of *A* to the radius,[21] the arm will vibrate in the same time as a pendulum whose length is 36.65, that is, in $\sqrt{\frac{36.65}{39.14}}$ seconds,[22] and therefore, if the stiffness of the wire is such as to make it vibrate in *N* seconds, the force which must be applied to each ball, in order to draw it aside by an angle *A*, is to the weight of the ball as the arch of $A \times \frac{1}{N^2} \times \frac{36.65}{39.14}$ to the radius. (Cavendish 1798, p. 509)

[21] What Cavendish is stating here is equivalent to saying that the force restoring the pendulum's motion (F_r) to the vertical through an angle *A* is to the weight of the ball times sin(*A*) (Falconer 1999, p. 475a).

[22] As in this case $\frac{x_1}{x_2}$ is proportional to $\frac{t_1^2}{t_2^2}$, it follows that $\sqrt{\frac{x_1}{x_2}}$ is proportional to $\frac{t_1}{t_2}$. If x_1 is 36.65 inches, x_2 is 39.14 inches and t_2^2 is 1, it follows that t_1 is proportional to $\sqrt{\frac{x_1}{x_2}}$ or, from what is given, proportional to $\sqrt{\frac{36.65}{39.14}}$.

Here Cavendish pointed out that the force exerted on the balls (F_e) swinging along a pendulum is to the restoring force (F_r) as $\frac{T^2}{N^2}$. Because the restoring force is, furthermore, proportional to the weight of the ball (W_b) times the arch of A, it follows that:

$$\frac{F_e}{W_b} :: \text{arch of } A \times \frac{T^2}{N^2} \left(= \text{arch of } A \times \frac{36.65}{39.14} \times \frac{1}{N^2} \right)$$

As the ivory scale at the end of the arm is 38.3 inches away from the center of motion and each division is $\frac{1}{20}$ of an inch from the center of motion, it subtends an angle at the center whose arch is $\frac{1}{766}$, i.e. $\frac{38.3\,\text{inch}}{0.05\,\text{inch}}$. Accordingly, we obtain that:

$$\frac{\text{The force which must be applied to each ball to draw it aside by one division}}{\text{The weight of the ball}} : \\ : \frac{1 \times 36.65}{766N^2 \times 39.14},$$

or,

$$\frac{\text{The force which must be applied to each ball to draw it aside by one division}}{\text{The weight of the ball}} :: \frac{1}{818N^2}. \quad (1)$$

By relying on the mathematical properties of the pendulum, Cavendish established a proportion that involves the periodic time.

> *Step 2*: Solving the proportion of the force of attraction of the weight on its corresponding ball to the force of attraction of the earth on that ball in terms of the mean density of the earth relative to the density of water.

Secondly, it is required to find "the proportion which the attraction of the weight bears to that of the earth thereon, supposing the ball to be place in the middle of the case, that is, to be not nearer to one side than the other" (ibid., p. 510). Each of the weights weighed 2,439,000 grains or roughly 158 kg,[23] which is equal to the weight of 10.64 spherical feet of water, i.e. equal to the weight of 10.64 times the volume of a sphere of water with a diameter of 1 foot.[24] The radius of one spherical feet of water is 6 inches, as 1 foot equals 12 inches. Therefore, Cavendish continued, the attraction of a weight on a particle placed at the center of a ball at 8.85 inches from the center of that weight (denoted as $F_{W \to b}^{8.85\,\text{inch}}$) is

[23] If we assume that 1 grain equals 64.79891 mg, then, the weight of weight W, W_W, corresponds to 158,044,541 mg or to 158 kg approximately.

[24] Allow me to explain Cavendish' statement. According to Cavendish, W_W equals 10.64 times the weight of a sphere of water with a diameter of 1 foot ($W_{\text{Sph.(1 foot)}}$): $W_W = 158$ kg $= 10.64 \times W_{\text{Sph.(1 foot)}}$. As $W_{\text{Sph.(1 foot)}} = V_{\text{Sph.(1 foot)}} \times \rho_{H_2O}$, we require the value which Cavendish introduced for the density of water, ρ_{H_2O}, and the volume of a spherical foot of water, $V_{\text{Sph.(1 foot)}}$. Since the diameter of a spherical foot of water is 1 foot and 1 foot equals 30.48 cm, its volume can be calculated, as follows: $V_{\text{Sph.(1 foot)}} = \frac{4}{3} \times \pi \times (15.24\,\text{cm})^3 = 1,483 \times 10\,\text{cm}^3 = 0.04183\,\text{m}^3$. In his paper, Cavendish did not explicitly provide a value for ρ_{H_2O}. However, the value he relied on can be found by working backwards. Since we know that: $158\,\text{kg} = 10.64 \times W_{\text{Sph.(1 foot)}} = 10.64 \times V_{\text{Sph.(1 foot)}} \times \rho_{H_2O} = 10.64 \times 0.01483\,\text{m}^3 \times \rho_{H_2O}$, it follows that: $\rho_{H_2O} = \frac{158\,\text{kg}}{10.64 \times 0.01483\,\text{m}^3 = 100 \times 10 \frac{\text{kg}}{\text{m}^3}}$,which agrees nicely with our present day value for ρ_{H_2O}. With this information at hand, we can make sense of Cavendish' claim that $W_W = 158$ kg $= 10.64 \times W_{\text{Sph.(1 foot)}}$. What we basically need is a solution for the ratio of W_W to $W_{\text{Sph.(1 foot)}}$. Since $W_W = 158$ kg and $W_{\text{Sph.(1 foot)}} = 0.01483\,\text{m}^3 \times 100 \times 10 \frac{\text{kg}}{\text{m}^3}$, it follows that the proportion of W_W to $W_{\text{Sph.(1 foot)}}$ is 10.7, which approximates Cavendish' value of 10.64.

to the attraction of a spherical foot of water on an equal particle placed on its surface (denoted as $F^{\text{surface}}_{\text{Sph.(1 foot)}\to b}$) as 10.64×0.9779[25] $\times \left(\frac{6}{8.85}\right)^2$ to 1, or:

$$\frac{F^{8.85\,\text{inch}}_{W\to b}}{F^{\text{surface}}_{\text{Sph.(1foot)}\to b}} :: \frac{10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2}{1}.$$

Furthermore, the mean diameter of the earth is 41,800,000 feet and, therefore, if the mean density of the earth is to that of water as D to 1,[26]

$$\frac{\text{The attraction of the weight on a ball}}{\text{The attraction of the earth on that same ball}} :: \frac{10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2}{41,800,000D},$$

or,

$$\frac{\text{The attraction of the weight on a ball}}{\text{The attraction of the earth on that same ball}} :: \frac{1}{8,739,000D} \tag{2}$$

During the derivation of (2), Cavendish omitted some crucial steps, which I will now fill in. Although Cavendish did not make this point explicit, this conclusion *was based on Newton's law of universal gravitation.*[27] This was of utter importance, for this application involves the mass of the earth, which is equal to the density of the earth times the volume of the earth. In other words, it is on the basis of the law of universal gravitation and the definition of mass that the (mean) density of the earth enters Cavendish' train of mathematical reasoning. Let $F_{W\to b}$ be the weight of W on ball x and $F_{E\to b}$ the weight of the earth on ball x, and let d refer to the diameter, ρ to the density, m to the mass, and r to the radius. Given the theory of universal gravitation, it follows that:

$$\frac{F_{W\to b}}{F_{E\to b}} :: 10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2 \times \frac{m_{(\text{Sph.(1 foot)})}}{r_{(\text{Sph.(1 foot)})^2}} \times \frac{rE^2}{mE},$$

so that:

$$\frac{F_{W\to b}}{F_{E\to b}} :: 10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2 \times \rho_{H_2O} \times \frac{d_{(\text{Sph.(1 foot)})^3}}{d_{(\text{Sph.(1 foot)})^2}} \times \frac{dE^2}{\rho_E \times dE^3}.$$ [28]

[25] At this point, Cavendish introduced a correction factor. He observed that, "[w]hen the weights are approached to the balls, their centres are 8.85 inches from the middle line of the case; but, through inadvertence, the distance, from each other, of the rods which support these weights, was made equal to the distance of the centres of the balls from each other, whereas it ought to have been somewhat greater." (Cavendish 1798, p. 510). As a consequence of this, the effect of the weights in drawing the arm aside is less than it would otherwise have been, to wit, in a ratio of 0.9779 to 1. This step follows from basic geometry. Since it is neatly described and illustrated in Mackenzie (1900, p. 89, footnote *), I will omit further discussion.

[26] Therefore D is proportional to the density of the earth divided by the density of water. As will be denoted here, $D = \frac{\rho_{\text{earth}}}{\rho_{H_2O}}$, or more succinctly, ρ_{E/H_2O}.

[27] Cf. Falconer (1999, p. 475b).

[28] Here Cavendish used the proportion $m :: \rho \times V$. As for a sphere, V is proportional to r^3, it follows that $m :: \rho \times r^3$. As r equals ½ d, $m :: \rho \times d^3$. The latter proportion justifies Cavendish' substitution of $m_{(\text{Sph.(1 foot)})}$ by $\rho_{H_2O} \times d_{(\text{Sph.(1 foot)})^3}$ and mE by $\rho_E \times dE^3$. As r equals ½ d, Cavendish is, furthermore, justified to substitute rE^2 by dE^2 and $r_{(\text{Sph.(1 foot)})^3}$ by $d_{(\text{Sph.(1 foot)})^3}$.

Now, since $d(_{\text{Sph.(1 foot)}})$ conveniently equals 1 inch,

$$\frac{F_{W\to b}}{F_{E\to b}} :: 10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2 \times \frac{1}{\rho_{(E/H_2O)}} \times \frac{1}{dE}.$$

Therefore,

$$\frac{F_{W\to b}}{F_{E\to b}} :: 10.64 \times 0.9779 \times \left(\frac{6}{8.85}\right)^2 \times \frac{1}{D} \times \frac{1}{41,800,000},$$

and, finally:

$$\frac{F_{W\to b}}{F_{E\to b}} \left(= \frac{\text{The attraction of the weight on a ball}}{\text{The attraction of the earth on that same ball}}\right) :: \frac{1}{8,739,000D},$$

which is indeed what Cavendish claimed in (2).

Step 3: Combining (1) and (2)

Previously, we have established that the force required to draw the arm through one division of 2.54 inch is to the weight of the ball as 1–818 N^2 (1). By dividing (1) and (2), we establish that:

$$\frac{\frac{\text{The force which must be applied to each ball to draw it aside by one division}}{\text{The weight of the ball}}}{\frac{\text{The attraction of the weight on a ball}}{\text{The attraction of the earth on that same ball}}} :: \frac{\frac{1}{818N^2}}{\frac{1}{8,739,000D}},^{29}$$

$$\frac{\text{The attraction of the weight on a ball}}{\text{The force which must be applied to each ball to draw it aside by one division}} :: \frac{N^2}{10,683D},$$

from which it follows that "the attraction [of the weight] will be able to draw the arm out of its natural position by […] $\frac{N^2}{10,683D}$ divisions." Therefore, "if on moving the weights from the midway to a near position the arm is found to move *B* divisions, or if it moves 2 *B* divisions on moving the weights form one near position to the other," the mean density of the earth relative to the density of water, *D*, is given by $\frac{N^2}{10,683B}$, where *B* is the number of divisions in hundredths of an inch and *N* is the observed period in seconds (Cavendish 1798, p. 511). By adding correction factors (1) and (4), which will be discussed in the accompanying footnote, to the above formula, Cavendish corrected $\frac{N^2}{10,683B}$ to $\frac{N^2}{10,844B}$ and by this expression he arrived at the values in column 7 in the table on Fig. 3 (ibid., p. 517).[30]

[29] Note that *the weight of the ball* = *the attraction of the earth on that ball.*

[30] Before Cavendish proceeded to compute the value of the density of the earth relative to the density of water on the basis of the values of *N* and *B* he had established in his experiments, Cavendish provided six correction factors: "[F]irst, for the effect which the resistance of the arm to motion has on the time of the vibration: 2d, for the attraction of the weights on the arm: 3d, for their attraction on the farther ball: 4th, for the attraction of the copper rods on the balls and arm: 5th, for the attraction of the case on the balls and the arm: and 6th, for the alteration of the attraction of the weights on the balls, according to the position of the arm, and the effect which that has on the time of vibration. None of these corrections, indeed, except the last, are of much signification, but they ought not entirely be neglected." (Cavendish 1798, p. 511). Cavendish computed that, (1) if the time of a vibration is given, the force required to draw the arm aside is greater than if the arm had no weight, namely in a proportion of 11,660 to 11,262, i.e. ca. 1.0353 to 1 (*first correction factor: the resistance of the arm to motion*; ibid., pp. 512–513), (2) that the power of the weight to move the

Familiarizing students with Cavendish' mathematical derivation for the mean density of the earth is no doubt a challenging exercise.[31] However it will help them to grasp the true significance and originality of the Cavendish Experiment, as mentioned above.

2.4 Cavendish' Determination of the Means Density of the Earth

Given his mathematical derivation of the mean density of the earth relative to the density of water, Cavendish could compute D on the basis of the experiments he performed with his torsion rod. The results of these computations are to be found in column 7 of Fig. 3.[32] In the concluding section to his paper, Cavendish recorded:

> From this table it appears, that though the experiments agree pretty well together, yet the difference between them, both in the quantity of motion of the arm and in the time of vibration, is greater than can proceed merely from the error of observation. As to the difference in the motion of the arm, it may very well be accounted for, from the current of air produced by the difference of temperature; but, whether this account for the difference in the time of vibration, is doubtful. If the current of air was regular, and of the same swiftness in all parts of the vibration of the ball, I think it could not; but

Footnote 30 continued
arm, by means of its attraction on the nearest part of it, is 0.0139 of its attraction on the ball (*second correction factor: the attraction of the weight on the arm*; ibid., pp. 513–514; Cavendish furthermore noted: "It must be observed, that the effect of the attraction of the weight on the whole arm is rather less than this, as its attraction on the farther half draws it the contrary way; but, as the attraction on this is small, in comparison of its attraction on the nearer half, it may be disregarded." (ibid., p. 514).), (3) that the effect of the attraction of the weight on both balls, is to that of its attraction on the nearest ball as 0.9983 to 1 (*third correction factor: the attraction on the farther ball*; ibid., p. 515), (4) that the attraction of the weight and copper rod on the arm and both balls together is to the attraction of the weight on the nearest ball as 1.0199–1 (*fourth correction factor: the attraction of the copper rods on the balls and arms*; ibid., p. 515), (5) that the attraction of the case on the balls cannot exceed $\frac{1}{1{,}170}$ part of the weight and that the whole force is "so small as not to be worth regarding" (*fifth correction factor: the attraction of the case and the balls and the arm*; ibid., pp. 515–517), and, finally, (6a) that "[i]f the time of vibration is determined by an experiment in which the weights are in near position, and the motion of the arm, by moving the weights from the *near to the midway position*, is d divisions, the *observed time* must be diminished in [...] in the ratio of $1 - \frac{d}{185}$ to 1; but when it is determined by an experiment in which the weights are in the midway position, no correction must be applied," (6b) that in order to correct "the motion of the arm caused by moving the weights *from a near to the midway position*, *or the reverse*, observe how much the position of the arm differs from 20 divisions, when the weights are in the near position: let this be n divisions, then, if the arm at that time is on the same side of the division of 20 as the weight, the *observed motion* must be diminished by the $\frac{2n}{185}$ part of the whole; but otherwise, it must be as much increased," and (6c) that "[i]f the weights are moved from *one near position to the other*, and the motion of the arm is $2d$ divisions, the *observed motion* must be diminished by the $\frac{2n}{185}$ part of the whole" (*sixth correction factor: the effect of the alternation of the attraction*; ibid., p. 519 [italics added]). Cavendish then added that "[i]f the weights are moved *from one near position to the other* (i.e., from – to + or from + to −), and the time of vibration is determined while the weights are in one of those positions, *there is no need of correcting either the motion of the arm, or the time of vibration*" (ibid. [italics added]). Because the balls oscillate in a non-uniform gravitational field due to the vicinity of the weights, the motion of the beam is not strictly harmonic and therefore the period of oscillation is affected; also, in their new equilibrium position, the balls' centers are no longer at a distance of 8.85 inch from the weights' centers, but at a smaller distance (Lauginie 2007, pp. 139–140). The great advantage of the symmetrical way of operation is that it allows errors due to an imperfect initial alignment to cancel in first order (ibid.).

[31] McCormmach observes: "He [i.e., Cavendish] covered the entire theoretical argument in a paragraph. He clearly thought it was obvious, as it may have been then. It isn't obvious to a reader today, but it is correct." (McCormmach 1998, p. 360).

[32] Let us, for instance, compute the value of D for entry 1, "+ to m." The time is equal to 14′55″ or 895 s and the motion of the arm expressed in twentieths of an inch is equal to 13.17. Accordingly, D equals $\frac{859^2}{13.17 \times 10{,}844} \approx 5.61$, which agrees with Cavendish' second result.

> as there will most likely be much irregularity in the current, it may very likely be sufficient to account for the difference. (ibid., p. 521)

Cavendish derived an average of 5.48 for the density of the earth relative to the density of water (ibid., p. 521).[33] The present-day value is ca. 5.517 (Lauginie 2007, p. 128). The accuracy, i.e. the closeness of an experimental result to the true value, with which Cavendish computed the mean density therefore remained within 1% of the present-day value (Jungnickel and McCormmach 2001, p. 450). Cavendish' average value of the density of the earth has been the object of debate (Baily 1843, pp. 90–91; Hutton 1821, pp. 289–290; Lauginie 2007, pp. 128–130). Note that Cavendish claimed that "*[b]y a mean of the experiments made with the wire first used* [i.e., the first six observations which were made with a "copper silvered" wire, which "was not stiff enough," as Cavendish explicitly pointed out (Cavendish 1798, p. 478)], *the density of the earth comes out 5,48 times greater than that of water*; and *by a mean of those made with the latter wire* [i.e., the remaining 23 observations, which were made with "a stiffer wire" (ibid., p. 485)], *it comes out the same*; and the extreme difference of the results of the 23 observations made with this wire is only, 75; so that the extreme results do not differ from the mean by more than, 38, or $\frac{1}{14}$ of the whole,[34] and therefore the density should seem to be determined hereby, to great exactness." (ibid., p. 521 [italics added]; cf. Lauginie 2007, p. 128). Using contemporary statistics it can be shown that the uncertainty in the mean, i.e. 5.48, amounts to 1.5% and the modern value 5.517 falls well within the uncertainty (Lauginie 2007, p. 140). Cavendish' claim with respect to the average of the first six observations is mistaken in the text: $\frac{5.5+5.61+4.88+5.07+5.26+5.55}{6}$ (see the first six values in column 7 of Fig. 3) equals ca. 5.31 and not ca. 5.48. However, if we replace the correct value 4.88[35] by a value of 5.88 in the same calculation, we arrive at an average of ca. 5.48 (Hutton 1821, p. 289; Baily 1843, p. 90; Lauginie 2007, pp. 128–129). Due to an inattentiveness on his part, Cavendish very likely introduced 5.88 instead of 4.88 when calculating the average of the first six experiments.[36] However, the mistake which Cavendish made when calculating the average value for D on the basis of the first six observations does not at all invalidate his calculation on the mean density as determined from observations 7–29 (Lauginie 2007, p. 130). Calculation shows that Cavendish' claim with respect to the average of observations 7–29 is entirely correct. Cavendish himself explicitly pointed out that observations 1–6 were made with a wire which "was not stiff enough"—thereby implicating that these observations have a weaker evidential significance than observations 7–29, which were made with "a stiffer wire." Correspondingly, "we must follow Cavendish himself who, very judiciously, considered separately each series of data (even if, by chance, he seemed to get the same mean result for both)" (ibid.).[37]

[33] Interestingly, in Proposition X of Book III, Newton had noted that "it is likely that the total amount of matter in the earth is about five or six times greater than it would be if the whole earth consisted of water" (Newton 1999, p. 815).

[34] 5.1 differs the most from the mean, namely by 0.38, which approximates $\frac{5.48}{14}$.

[35] By computing the relevant data, i.e. by taking $N = 14'42''$ (or 882 s) and $B = 14.69$, we find that $D = \frac{882^2}{14.69 \times 10{,}844} \approx 4.88$.

[36] It is difficult to unearth the exact circumstances of Cavendish's error, because no manuscripts on the Cavendish experiment have survived.

[37] Given Cavendish' careful distinction between the two series, it makes little sense to take the mean of all 29 experiments as some authors have done (Baily 1843, p. 90; Danson 2005, p. 248).

3 The Aftermath of the Cavendish Experiment and its Educational Significance

In discussing the aftermath of the Cavendish' Experiment, I shall, given my current agenda, focus on certain experiments and on certain aspects of these experiments.[38] We will start by looking at nineteenth-century determinations of the mean density of the earth in Sect. 3.1. Thereafter, in Sect. 3.2, we will look at late nineteenth-century determinations of the gravitational constant. Finally, in Sect. 3.3, I shall go into the evidential significance of the material discussed in Sect. 3.1 and Sect. 3.2.

3.1 Nineteenth-Century Determinations of the Mean Density of the Earth

In 1843 Francis Baily urged that Cavendish' aim in his paper on the mean density of the earth "appears to have been more for the purpose of exhibiting a *specimen* of what he considered to be an excellent method of determining this important inquiry, than of deducing a result that should lay claim to the full confidence of the scientific world" (Baily 1843, p. 8). Correspondingly, he emphasized that Cavendish' results were approximate and few in number. In order to improve Cavendish' determination of the mean density of the earth, the Royal Society had set out "not merely to repeat the original experiments of Cavendish in a somewhat similar manner, but also to extend the investigation by varying the magnitude and substance of the attracted balls—by trying the effect of different modes of suspension—by adopting considerable differences of temperature—and by other variations that might be suggested during the progress of inquiry" (ibid., p. 10). Baily used a so-called 'inverted' torsion rod: whereas Cavendish had suspended the weights from above, Baily reversed the *modus operandi*. An inverted T-shaped mahogany box, which contained the suspended torsion rod, was attached to the ceiling by "a very stout plank" (ibid., p. 11). Baily recorded that "[t]he whole of the mahogany box is completely insulated from every part of the frame work, and from any contact with those portions of the apparatus that are near it. It consequently remains undisturbed either by walking about the floor, by working the masses, or by any other commotion within the room." (ibid., p. 20). Below the centre of the mahogany box, a solid wooden piece was firmly screwed to the floor, "on which has been raised a circular frame work, embracing and supporting a copper ring; within which ring a large round wooden pillar moves on an iron pivot, which bears upon a small metal cup" (ibid., pp. 11–12). On top of the pillar, a deal plank was fastened horizontally, which supported two large leaden balls or masses, which were firmly fixed onto it (ibid., p. 12). In order to minimize the "influence of any accidental or sudden change of temperature [i.e., as Cavendish has pointed out, the most significant source of error] in the room," an octagonal wooden frame was built around the horizontal portion of the mahogany box and the support of the leaden balls (ibid., p. 13).[39] Accordingly, Baily made sure that the surfaces of the masses could not approach the torsion box nearer than about an inch, "conceiving that this increased distance would be a sufficient protection,"

[38] See Ducheyne (2011) for a detailed overview.

[39] Baily noted that Cavendish' experimental set-up suffered from the unprotected state of the torsion box: "[i]n both cases the masses were brought up almost *close* to the outer side of this wooden shaft, but without actually touching it: but no mention is made of the application of any intervening substance to guard against a change of temperature on the approach of the masses" (Baily 1843, p. 35).

(ibid., pp. 38–39) and also gilded the masses "for the purpose of preventing the effect of [heat] radiation, from whatever source it might arise" (ibid., p. 41). Baily remarked that "[n]othing can exceed the ease, the steadiness, and the facility with which these large bodies are moved: and during the many thousands of times that they have been turned backwards and forwards, I have never observed the least deviation from the most perfect accuracy" (ibid., p. 15). Baily also emphasized that during his experiments he has never observed any irregular lateral or angular motion in the torsion rod (ibid., pp. 30–31). In a period of 18 months, Baily performed nearly 1,300 experiments, of which some 1,000 experiments were effectively used. Baily, nevertheless, admitted that discordances occasionally occur, "which cannot wholly be attributed to change of temperature, but to some other occult influence with which we are at present unacquainted" (ibid., p. 44).

As we have seen, Baily tried to improved upon Cavendish' procedures of eliminating or minimizing external disturbances, especially temperature variations. The mean result of Baily's large body of experiments gave a value of 5.6747 for the mean density of the earth, which differs ca. 3% from the present-day value, with a probable error of 0.0038, which provides a value more precise that Cavendish' (ibid., ccxlvii). It turns out, however, that Baily's result was—somewhat ironically—less accurate than Cavendish': the commonly accepted value for the mean density of the earth (5.517) does not fall within the margin of error of Baily's value, while it does fall within the margin of error of Cavendish', as was highlighted in Sect. 2.4. In the 1870s, two French physicists, Marie-Alfred Cornu and Jean-Baptistin Baille, accounted for Baily's somewhat off-track measurement. They pointed out that Baily did not sufficiently take into account a systematic error caused by the inversion of the attracting weights on their pivot, which produced some minute trepidations (Cornu 1878, p. 701). Once the required correction was applied to Baily's result, Cornu and Baille obtained 5.55 as the corrected value for the mean density of the earth (ibid., pp. 701–702), which differs ca. 0.6% from the present-day value.

This episode in the history of nineteenth-century research on the mean density of the earth shows that physicists were seeking ways to isolate the gravitational interaction between laboratory-sized bodies, on the basis of which they determined the mean density of the earth, by reflecting on the *modi operandi* followed by their predecessors. The discussion of Baily's experiments also indicates that realizing this endeavor was not always a process of uniform progress: Baily's result was less accurate than Cavendish'. It was only by the end of the nineteenth century that Cavendish' determination of the mean density of the earth was significantly improved upon as will be shown in Sect. 3.2.

3.2 Nineteenth-Century Determinations of the Gravitational Constant

As we have seen in Sect. 1, the mean density of the earth and the gravitational constant are easily derivable from each other: the former is given by $\frac{g \cdot R(e)^2}{G \cdot V(e)}$; the latter by $\frac{g \cdot R(e)^2}{\rho(e) \cdot V(e)}$. Moreover, both physical quantities were determined from the gravitational interaction between laboratory-sized bodies. Near the end of the nineteenth century, the focus of research had shifted: the quest for G now occupied center stage, while the determination of the mean density of the earth was conceived of as a straightforward exercise once G had been determined. Scientists active in this field moved away from performing such "purely local experiments" as determining the mean density of the earth and, since they were determining the exact expression of the law of universal gravitation, they now considered themselves as "working for the universe" (cf. Poynting 1920, p. 633). In what follows, I shall touch upon John H. Poynting's determination of the gravitational constant by way of example. I shall also briefly mention the results obtained by Charles V. Boys, Carl Braun, and Franz Richarz and Otto Krigar-Menzel (Boys 1895; Braun 1896; Richarz and Krigar-Menzel 1898).[40]

In order to determine *G*, Poynting suspended two nearly equal masses *A* and *B* (21,582.33 and 21,566.21 g, respectively) from a balance (Poynting 1892, p. 579). *A* and *B* were, furthermore, placed within a wooden case. Next, mass *M*, weighing 153,407.26 g, was placed underneath *A*. Once the change in position of the beam has been observed, which has a length of 1.23329 m (ibid., p. 571), *M* is turned 180° degrees so that it is underneath *B*. Thereafter, the position of the beam is observed once again. As *M* switches sides from *A* to *B*, the attraction is taken away from *A* and added to *B*. To eliminate the attraction of *M* on the beam and the suspending wires, *A* and *B* are raised to the equally higher positions A' and B', "[f]or the *difference* between the two increments of weight on the right, is due solely to the alteration of the positions of *A* and *B* relative to *M*, the attraction on the beam remaining the same in each" (ibid., p. 567). In order to compensate for the tilting of the floor which arises when *M* is moved, an additional mass *m*, which is nearly half as big as *M* (namely, 76,497.4 g), was installed twice as far from the axis and on the opposite side of *M* (ibid., pp. 567–568, p. 579). Due to the addition of *m*, the "resultant pressure was now always through the axis" and no "tilting of the floor when the turntable was moved" could be detected (ibid., p. 569). Both *M* and *m* were steadily placed on a turntable, which could be manipulated in the room above the basement, in which the apparatus was installed. A scale was, furthermore, fixed horizontally to the end of the telescope by means of which the subsidiary riders attached to the centre of the balance beam could be monitored, and hence the tilt of the beam. During the experiment, air currents and variations in temperature and air pressure were avoided as good as possible. Upon processing the obtained data, Poynting established a value of *G* of $6.6984 \times 10^{-8} \frac{\text{cm}^3}{\text{g} \times \text{s}^2}$ (or $6.6984 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}$) (ibid., p. 612). Once *G* was determined, Poynting concluded that the mean density relative to the density of water is equal to 5.4934 (ibid., p. 607).

Three years after the publication of Poynting's paper, Boys published a new method for determining *G*. Upon computing the data obtained, Boys inferred that the value for *G* is equal to $6.6579 \times 10^{-8} \frac{\text{cm}^3}{\text{g} \times \text{s}^2}$ (or $6.6579 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}$) (Boys 1895, p. 62). From this he obtained a value of 5.5268 for the mean density of the earth. One year after the publication of Boys' paper, Braun published a new determination of the gravitational constant. On the basis of a totally different experimental apparatus, Braun established a value of $6.65816 \pm 0.00168 \times 10^{-8} \frac{\text{cm}^3}{\text{g} \times \text{s}^2}$ (or $6.65816 \pm 0.00168 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}$) for the gravitational constant; correspondingly, he arrived at a value of 5.52700 ± ca. 0.0014 for the mean density of the earth relative to the density of water (Braun 1896, p. 258*c*). Finally, in 1898 Richarz and Krigar-Menzel published the results they obtained from experimenting with yet another device. In their joint paper, they concluded that the gravitational constant equals $6.685 \pm 0.011 \times 10^{-8} \frac{\text{cm}^3}{\text{g} \times \text{s}^2}$ (or $6.685 \pm 0.011 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}$) and, on the basis of this determination, they arrived at a value of 5.505 ± 0.009 for the mean density of the earth relative to the density of water (Richarz and Krigar-Menzel 1898, p. 110).

In these late nineteenth-century experiments, additional non-gravitational factors were screened-off (e.g., atmospheric pressure and atmospheric humidity) and variations in

[40] I refer the interested reader to Ducheyne (2011, pp. 208–218), in which Poynting's, Boys', Braun's, and Richarz and Krigar-Menzel's experiments and results are discussed in more detail.

temperature and air currents were screened-off more stringently. By eliminating non-gravitational disturbances more successfully, physicists became more successful in isolating the gravitational interaction between laboratory-sized bodies, on the basis of which they could determine big *G*. Accordingly, they established increasingly converging and increasingly accurate measurements of big *G* and the density of the earth.[41]

3.3 Discussion

The material surveyed in subsections 3.1 and 3.2 provides an excellent occasion to discuss the evidential significance of so-called 'normal science'. As is widely known, Thomas S. Kuhn stressed that the empirical work undertaken to articulate a paradigm theory consists in "resolving some of its residual ambiguities and permitting the solution of problems to which it had previously only drawn attention" (Kuhn 1996, p. 27). Conceiving of 'normal science' as mere puzzle-solving or as straightforward theory application comes at a risk of underestimating the evidential significance of the eighteenth- and nineteenth-century gravitational research which I have addressed in this paper. That Kuhn indeed minimized the evidential significance of so-called 'normal science' can be seen from the following quote, in which he wrote:

> *Exploring the agreement between theory and experiment into new areas or to new limits of precision* is a difficult, unremitting, and, for many, exciting job. Though *its object is neither discovery nor confirmation*, its appeal is quite sufficient to consume almost the entire time and attention of those physical scientist who do quantitative work." (Kuhn 1961, p. 174 [italics added]).

According to Kuhn, physical scientists doing "quantitative work" are not involved in confirming the theory on the basis of which they establish theory-mediated measurements.

To impose a gulf between "quantitative work" and confirmation and to claim that normal science has no evidential significance is, however, misleading. In the course of this paper, I have shown that physicists measured the mean density of the earth and the gravitational constant from the gravitational attraction between laboratory-sized bodies. In order to establish reliable determinations of the mean density of the earth and big *G*, scientists were indeed highly concerned with pushing back external disturbances and with producing "a more stable, less noisy version of the [same] phenomenon" (Hacking 1983, p. 231). That laboratory-sized bodies are impelled towards one another under increasingly stringently screened-off experimental conditions provided increasingly stronger evidence that the coming together of laboratory-sized bodies is not due to non-gravitational disturbances, but to be ascribed to the gravitational interaction between those bodies as stipulated by the theory of universal gravitation. In other words, increasingly stronger evidence was established for the robustness of the gravitational interaction between laboratory-sized bodies, and, hence, for the *universality* of the theory of gravitation. The Cavendish Experiment and its aftermath therefore provides an excellent opportunity to explain to students that 'normal science' encompasses more than puzzle-solving or theory application: it involves continuous and increasingly stringent tests of the paradigmatic theory at hand, which, if successful, results in increasingly stronger evidence that supports that theory.

[41] The percentages by which the preceding measurements of *G* differ from the present-day value, i.e. $6.7384 \pm 0.00012 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \times \mathrm{s}^2}$, are approximately as follows: 0.6% (Poynting), 1.2% (Boys and Braun), and 0.8% (Richarz and Krigar-Menzel). The percentages by which they differ from the present-day value for the mean density of the earth, i.e. 5.517, are approximately: 0.43% (Poynting), 0.18% (Boys and Braun), and 0.22% (Richarz and Krigar-Menzel).

4 Conclusion

In Sects. 2 and 3, I have shown that teaching a historically sensitive reconstruction of the Cavendish Experiment has at least three educational benefits. Although studying the Cavendish Experiment and its aftermath will no doubt be challenging and demanding for students and teachers alike, much can be learned from it on the role of equations in physics, the role of experimentation and measurement in physics, the evidential significance of 'normal science', and the general development of physics.

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